JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 39, No. 5, September–October 2002

Satellite Attitude Stabilization Through Kite-Like Tether Configuration

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A novel method for attitude stabilization of satellites using flexible tethers is presented. The feasibility of achieving desired attitude control of a satellite is explored. The proposed controller model assumes a downward deployment of a small auxiliary mass from the satellite through a kite-like tether configuration. The Lagrangian formulation approach is utilized to develop the governing system of nonlinear ordinary differential equations for the proposed constrained system. Results of numerical simulations of the nonlinear governing equations of motion and a stability analysis of motion-in-the-small undertaken indicate the feasibility of achieving desired attitude control. Furthermore, the stability analysis leads to useful design criteria in the form of inequality constraints on the system parameters. Finally, the attitude performance of the proposed system is compared with one-tether and two-tether systems.

Nomenclature

$\hat{a}, \hat{b}$	=	$\hat{a}_1$ or $\hat{a}_2$ when $\hat{a}_1 = \hat{a}_2$; $\hat{b}_1$ or $\hat{b}_2$ when $\hat{b}_1 = \hat{b}_2$
a_i, b_i	=	z, x coordinates or horizontal, vertical offsets of tether attachment points A ($i = 1$) and B ($i = 2$) in the satellite S-$x_0 y_0 z_0$ coordinate frame, m
$\hat{a}_i, \hat{b}_i$	=	a_i/L_{ref}, b_i/L_{ref}
C	=	$EA/(m_2 L_{\mathrm{ref}} \Omega^2)$
Cj	=	$C\{(1-e^2)/(1+e\cos\theta)\}^3 \varepsilon_j U_j$, $j = 1, 2, 3$
c_ζ	=	cosine of angle ζ
D	=	tether junction
EA	=	tether modulus of rigidity, N
e	=	orbital eccentricity
f_i	=	ith constraint function
I_k	=	satellite principal centroidal moments of inertia about k axis, $k = x, y, z$, kg-m^2
I_r	=	satellite mass distribution parameter; $(I_y - I_z)/I_x$
L	=	distance between satellite mass center and tether junction, that is, distance SD, m
L_e	=	value of L at system equilibrium, m
L_j	=	stretched jth tether length, m
L_{j0}	=	unstreched jth tether length, m
L_{ref}	=	reference length; $(I_x/m_2)^{1/2}$, m
L_0	=	L when tether strain is zero, m
L_4	=	distance between satellite mass center and auxiliary mass, that is, distance SE, m
L_{40}	=	L_4 at $\theta = 0$, that is, $L_0 + L_{30}$, m
l	=	L/L_{ref}
l_e	=	L_e/L_{ref}
l_j	=	L_j/L_{ref}
l_{j0}	=	L_{j0}/L_{ref}
l_{r0}	=	l_{10} or l_{20} when $l_{10} = l_{20}$
l_0	=	L_0/L_{ref}
l'_0	=	l' at $\theta = 0$
$(l_{30})_{\mathrm{cr}}, (l_{r0})_{\mathrm{cr}}$	=	critical or minimum dimensionless tether length l_{30} and l_{r0}, respectively
l_{40}	=	L_{40}/L_{ref}
l'_{40}	=	l'_4 at $\theta = 0$
m_1	=	satellite mass, kg
m_2	=	auxiliary mass, kg
Q	=	generalized force corresponding to generalized coordinate q
R	=	orbital radius, m
R_a	=	semimajor axis of the system center of mass, m
S	=	satellite mass center
S-xyz	=	satellite body coordinate
S-$x_t y_t z_t$	=	coordinate frame describing $\boldsymbol{L}$
S-$x_0 y_0 z_0$	=	coordinate axes in the local vertical frame
T	=	kinetic energy of the system, N-m
u_j	=	unit function, 1 for $\varepsilon_j \geq 0$ and 0 for $\varepsilon_j < 0$
V	=	potential energy of the system, N-m
α	=	satellite pitch angle, deg
α_0	=	α at $\theta = 0$, deg
α'_0	=	α' at $\theta = 0$
β	=	in-plane swing angle of $\boldsymbol{L}$, deg
β_e	=	β at system equilibrium, deg
β_0	=	β at $\theta = 0$, deg
β'_0	=	β' at $\theta = 0$, deg
γ	=	angle between the line SE and $\boldsymbol{L}$, deg
γ_0	=	γ at $\theta = 0$, deg
γ'_0	=	γ' at $\theta = 0$, deg
ε_j	=	jth tether strain
ζ	=	angle between tethers AD and BD at tether junction, deg
θ	=	true anomaly as measured from the reference line, deg
Λ_i	=	Lagrange multiplier for ith constraint
λ_i	=	$[1/(m_2 L_{\mathrm{ref}}^2 \Omega^2)][(1-e^2)/(1+e\cos\theta)]^3 \Lambda_i$, $i = 1, 2, \ldots, 7$
μ	=	Earth's gravitational constant, m^3/s^2
ϕ	=	in-plane swing angle of $\boldsymbol{L}_3$, deg
ϕ_0	=	ϕ at $\theta = 0$, deg
ϕ'_0	=	ϕ' at $\theta = 0$, deg
Ω	=	mean angular velocity $(\mu/R_a^3)^{1/2}$, rad/s
$(\cdot)_{\mathrm{cr}}$	=	critical or minimum value of $(\cdot)$
$(\cdot)_j$	=	$(\cdot)$ for jth tether, $j = 1, 2, 3$
$(\cdot)_0$	=	$(\cdot)$ at $\theta = 0$
$(\cdot)'$, $(\cdot)''$	=	$\mathrm{d}(\cdot)/\mathrm{d}\theta$ and $\mathrm{d}^2(\cdot)/\mathrm{d}\theta^2$, respectively
$\lvert(\cdot)\rvert_{\max}$	=	absolute maximum amplitudes of $(\cdot)$

Introduction

THE advent of tethered satellite systems (TSS)[1] marks the beginning of a new era in space research. Several interesting space applications of tethers[2] have been proposed and analyzed. The earlier methods suggested for ensuring satellite librational stability

Received 6 December 2001; revision received 30 April 2002; accepted for publication 1 May 2002.

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through a single tether-mass attachment were based on feedback control of the control moment by regulating tether lengths or offsets alone, or in combination with established active control devices.[1–6] Some of the contemporary investigations focused on the deployment of a subsatellite mass through two or more tethers. Misra and Diamond[7] proposed a TSS model composed of a subsatellite and a main satellite body connected by two extensible, but massless tethers. They considered the TSS motion in a circular orbit along with tether in-plane motion, out-of-plane motion, and longitudinal oscillations. Ciardo and Bergamaschi[8] considered three-dimensional attitude motion of the two-tether system. They considered tether in-plane and out-of-plane swing angles without tether tension variations. They used linearized equations of motion to obtain simulation results. Banerjee and Kane[9] proposed the concept of pulling on tethers to control the pitching stability of an auxiliary space platform connected by two tethers to a space station. Kumar[10] studied the attitude stabilization of the satellite moving in a circular orbit using two tethers. The case of such a satellite moving in elliptic orbits was considered by Kumar and Kumar,[11,12] and they showed that the achievement of nearly passive satellite pointing stability by using two tethers appears feasible.

It is found from the preceding research that the most simple tether configuration of a single tether connecting the main satellite to the auxiliary mass requires feedback control to achieve desired attitude control of the satellite and that the two-tether system provides better satellite attitude precision compared to one-tether system. However, the two-tether system is more complex, and, as it involves two long tethers, there exists a possibility of their entanglements. Therefore, in order to solve these problems and take the benefit of two-tether configuration, the present study on the proposed kite-like tether configuration was undertaken.

The Lagrangian formulation procedure is utilized to obtain the governing ordinary differential equations of motion for the proposed constrained system moving in an elliptic orbit. Because of the relatively short tether length considered in this study, it is assumed that the tether dynamics does not affect the orbital dynamics. Next, a first-order pitching stability analysis is undertaken to determine the necessary design limits on system parameters. Finally, for a detailed assessment of the proposed attitude control strategy the set of exact governing equations of motion is numerically integrated.

Proposed Satellite Attitude Controller Model and Equations of Motion

The investigation is initiated by formulating the equations of motion of the proposed satellite controller moving in an elliptic orbit. The proposed controller model assumes a downward deployment of a small auxiliary mass from the satellite through a kite-like tether configuration (Fig. 1). Two identical tethers are attached to the satellite at two distinct points symmetrically offset from its mass center and below the satellite's principal z axis. The other ends of the two tethers are connected to a tether deploying the auxiliary mass in kite-like configuration.

The auxiliary mass m_2 is assumed to be much smaller than the main satellite mass m_1 and treated as particle. The tethers, made of a light material like Kevlar®, are assumed to have negligible mass. Their damping effects and transverse vibrations are ignored. To facilitate analytical treatment of the problem only, the case involving in-plane angular motion has been investigated. The satellite is assumed to be vertically above the ground station while passing over the ascending node. The corresponding nodal line represents the reference line in orbit for the measurement of the true anomaly. The coordinate frame $x_0y_0z_0$ passing through the system center of mass S represents the orbital reference frame. Here, the x_0 axis is taken along normal to the orbital plane, y_0 axis points along the local vertical, and z_0 axis represents the third axis of this right-handed frame. The orientation of the satellite is specified by a rotation α (pitch) about the x axis. The corresponding satellite principal body-fixed coordinate frame is denoted by S-xyz. For the variable length $\boldsymbol{L}$ joining the satellite mass center S and tether junction D, the angle β denotes rotation about the axis normal to the orbital plane and is referred as the in-plane swing angle. The resulting coordinate frame associated with this vector is denoted by S-$x_t y_t z_t$. The system under

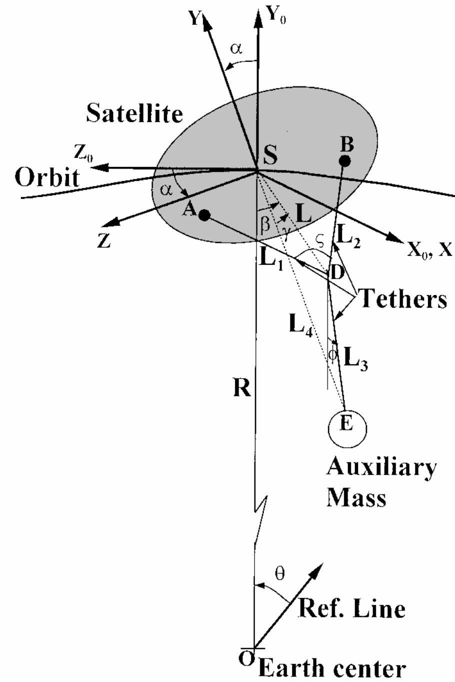

Fig. 1 Geometry of kite-like tethered satellite configuration.

consideration has 10 generalized coordinates: satellite pitch rotation (α), $\boldsymbol{L}$ (length L and angle β), tether strains (ε_j, $j = 1, 2, 3$), c_ζ, γ, ϕ, and L_4. For convenience of system representation and response simulation, the governing relations are expressed in dimensionless form. The preceding generalized variables are not independent because they are related through dimensionless constraints as follows:

$$f_1 = l_1 - \left[\hat{a}_1^2 + \hat{b}_1^2 + l^2 - 2\hat{a}_1 l \sin(\alpha - \beta) - 2\hat{b}_1 l \cos(\alpha - \beta)\right]^{\frac{1}{2}} = 0$$

$$f_2 = l_2 - \left[\hat{a}_2^2 + \hat{b}_2^2 + l^2 + 2\hat{a}_2 l \sin(\alpha - \beta) - 2\hat{b}_2 l \cos(\alpha - \beta)\right]^{\frac{1}{2}} = 0$$

$$f_3 = 2 l_1 l_2 c_\zeta - l_1^2 - l_2^2 + (\hat{a}_1 + \hat{a}_2)^2 = 0$$

$$f_4 = \varepsilon_3 u_3 - \left[(\varepsilon_1 u_1)^2 + (\varepsilon_2 u_2)^2 + 2\varepsilon_1 u_1 \varepsilon_2 u_2 c_\zeta\right]^{\frac{1}{2}} = 0$$

$$f_5 = \varepsilon_3 u_3 \sin\phi - \varepsilon_1 u_1 (\hat{a}_1 \cos\alpha - \hat{b}_1 \sin\alpha + l \sin\beta)/l_1 - \varepsilon_2 u_2 (-\hat{a}_2 \cos\alpha - \hat{b}_2 \sin\alpha + l \sin\beta)/l_2 = 0$$

$$f_6 = [l + l_3 \cos(\beta - \phi)] \tan\gamma - l_3 \sin(\beta - \phi) = 0$$

$$f_7 = l_4 - l \cos\gamma - \left(l_3^2 - l^2 \sin^2\gamma\right)^{\frac{1}{2}} = 0 \qquad (1)$$

where $l_j = l_{j0}(1 + \varepsilon_j)$, $j = 1, 2, 3$; $c_\zeta = \cos\zeta$.

To apply the Lagrangian approach for the formulation of the system equations of motion, the expressions for the system kinetic energy T as well as the potential energy V are first obtained:

$$T = \frac{1}{2}(m_1 + m_2)(\dot{R}^2 + \dot{\theta}^2 R^2) + \frac{1}{2} m_2 \left[\dot{L}_4^2 + (\dot{\theta} + \dot{\beta} - \dot{\gamma})^2 L_4^2\right] + \frac{1}{2} I_x (\dot{\theta} + \dot{\alpha})^2$$

$$V = -\frac{\mu}{R}(m_1 + m_2) + \frac{1}{4}\frac{\mu}{R^3}\{(I_x + I_y + I_z) - 3[I_x + (I_z - I_y)\cos 2\alpha]\} + \frac{1}{2}\frac{\mu}{R^3} m_2 [1 - 3\cos^2(\beta - \gamma)] L_4^2 + \frac{1}{2} EA \sum_{j=1}^{3} L_{j0} \varepsilon_j^2 u_j \qquad (2)$$

The term u_j in the potential energy expression is simply a unit function, the use of which precludes any negative strain in the tether.

The Lagrangian equations of motion corresponding to the various generalized coordinates indicated earlier can be obtained using the general relation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = Q + \sum_{i=1}^{7} \Lambda_i \frac{\partial f_i}{\partial q} \tag{3}$$

where q is the generalized coordinate.

By substituting the generalized coordinates in the preceding equations and carrying out the algebraic manipulation and nondimensionalization, we get the following governing nonlinear, coupled ordinary differential equations of motion in the dimensionless form.

Satellite pitch (α):

$$\begin{aligned}&(1+e\cos\theta)\alpha'' - 2(1+\alpha')e\sin\theta - 1.5I_r\sin(2\alpha)\\&\quad-\lambda_1[\hat{a}_1 l\cos(\alpha-\beta) - \hat{b}_1 l\sin(\alpha-\beta)]/l_1\\&\quad+\lambda_2[\hat{a}_2 l\cos(\alpha-\beta) + \hat{b}_2 l\sin(\alpha-\beta)]/l_2\\&\quad-\lambda_5[-\varepsilon_1 u_1(-\hat{a}_1\sin\alpha - \hat{b}_1\cos\alpha)/l_1\\&\quad-\varepsilon_2 u_2(\hat{a}_2\sin\alpha - \hat{b}_2\cos\alpha)/l_2] = 0\end{aligned} \tag{4}$$

Vector L, in-plane swing (β):

$$\begin{aligned}&(1+e\cos\theta)(\beta''-\gamma'') - 2[1+(\beta'-\gamma')]e\sin\theta\\&\quad+1.5\sin[2(\beta-\gamma)] + 2(1+e\cos\theta)(1+\beta'-\gamma')l_4'/l_4\\&\quad-\{-\lambda_1[\hat{a}_1 l\cos(\alpha-\beta) - \hat{b}_1 l\sin(\alpha-\beta)]/l_1\\&\quad+\lambda_2[\hat{a}_2 l\cos(\alpha-\beta) + \hat{b}_2 l\sin(\alpha-\beta)]/l_2\\&\quad-\lambda_5(\varepsilon_1 u_1/l_1 + \varepsilon_2 u_2/l_2) l\cos\beta\\&\quad-\lambda_6 l_3[\tan\gamma\sin(\beta-\phi) + \cos(\beta-\phi)]\}\big/l_4^2 = 0\end{aligned} \tag{5}$$

Vector L, dimensionless length (l):

$$\begin{aligned}&-\lambda_1[l - \hat{a}_1\sin(\alpha-\beta) - \hat{b}_1\cos(\alpha-\beta)]/l_1 - \lambda_2[l + \hat{a}_2\sin(\alpha-\beta)\\&\quad-\hat{b}_2\cos(\alpha-\beta)]/l_2 - \lambda_5(\varepsilon_1 u_1/l_1 + \varepsilon_2 u_2/l_2)\sin\beta\\&\quad+\lambda_6\tan\gamma + \lambda_7(l - l_4\cos\gamma)/(l_4 - l\cos\gamma) = 0\end{aligned} \tag{6}$$

Tether strain (ε_1):

$$\begin{aligned}&C_1 l_{10} - \lambda_1 l_{10} - 2\lambda_3 l_{10}(c_\zeta l_2 - l_1) + \lambda_4 u_1(\varepsilon_1 + c_\zeta\varepsilon_2 u_2)/(\varepsilon_3 u_3)\\&\quad+\lambda_5 l_{10} u_1(\hat{a}_1\cos\alpha - \hat{b}_1\sin\alpha + l\sin\beta)\big/l_1^2 = 0\end{aligned} \tag{7}$$

Tether strain (ε_2):

$$\begin{aligned}&C_2 l_{20} - \lambda_2 l_{20} - 2\lambda_3 l_{20}(c_\zeta l_1 - l_2) + \lambda_4 u_2(\varepsilon_2 + c_\zeta\varepsilon_1 u_1)/(\varepsilon_3 u_3)\\&\quad+\lambda_5 l_{20} u_2(-\hat{a}_2\cos\alpha - \hat{b}_2\sin\alpha + l\sin\beta)\big/l_2^2 = 0\end{aligned} \tag{8}$$

Cosine of angle $\zeta{:}c_\zeta$:

$$2\lambda_3 l_1 l_2 - \lambda_4\varepsilon_1 u_1\varepsilon_2 u_2/(\varepsilon_3 u_3) = 0 \tag{9}$$

Tether strain (ε_3):

$$\begin{aligned}&C_3 l_{30} - \lambda_4 u_3 - \lambda_5 u_3\sin\phi - \lambda_6 l_{30}[\tan\gamma\cos(\beta-\phi) - \sin(\beta-\phi)]\\&\quad+\lambda_7 l_3 l_{30}/(l_4 - l\cos\gamma) = 0\end{aligned} \tag{10}$$

Angle ϕ:

$$\lambda_5\varepsilon_3 u_3\cos\phi + \lambda_6 l_3[\tan\gamma\sin(\beta-\phi) + \cos(\beta-\phi)] = 0 \tag{11}$$

Angle γ:

$$\begin{aligned}&(1+e\cos\theta)(\beta''-\gamma'') - 2[1+(\beta'-\gamma')]e\sin\theta\\&\quad+1.5\sin[2(\beta-\gamma)] + 2(1+e\cos\theta)(1+\beta'-\gamma')l_4'/l_4\\&\quad+\big\{-\lambda_6[l + l_3\cos(\beta-\phi)]\big/\cos^2\gamma\\&\quad+\lambda_7 l l_4\sin\gamma/(l_4 - l\cos\gamma)\big\}\big/l_4^2 = 0\end{aligned} \tag{12}$$

Dimensionless length $L_4(l_4)$:

$$\begin{aligned}&(1+e\cos\theta)l_4'' - 2l_4' e\sin\theta - l_4(1+e\cos\theta)(1+\beta'-\gamma')^2\\&\quad+l_4[1 - 3\cos^2(\beta-\gamma)] - \lambda_7 = 0\end{aligned} \tag{13}$$

where

$$\lambda_i = \left[1\big/\left(m_2 L_{\mathrm{ref}}^2\Omega^2\right)\right][(1-e^2)/(1+e\cos\theta)]^3\Lambda_i \qquad i = 1, 2, \ldots, 7$$

$$Cj = C[(1-e^2)/(1+e\cos\theta)]^3\varepsilon_j U_j, \qquad j = 1, 2, 3$$

By solving Eqs. (7–13), λ_i, $i = 1, 2, \ldots, 7$ are obtained as follows:

$$\begin{aligned}\lambda_1 &= C_1 - 2\lambda_3(c_\zeta l_2 - l_1) + \lambda_4 u_1(\varepsilon_1 + c_\zeta\varepsilon_2 u_2)/(l_{10}\varepsilon_3 u_3)\\&\quad+\lambda_5 u_1(\hat{a}_1\cos\alpha - \hat{b}_1\sin\alpha + l\sin\beta)\big/l_1^2\end{aligned} \tag{14}$$

$$\begin{aligned}\lambda_2 &= C_2 - 2\lambda_3(c_\zeta l_1 - l_2) + \lambda_4 u_2(\varepsilon_2 + c_\zeta\varepsilon_1 u_1)/(l_{20}\varepsilon_3 u_3)\\&\quad+\lambda_5 u_2(-\hat{a}_2\cos\alpha - \hat{b}_2\sin\alpha + l\sin\beta)\big/l_2^2\end{aligned} \tag{15}$$

$$\lambda_3 = \lambda_4\varepsilon_1 u_1\varepsilon_2 u_2/(2l_1 l_2\varepsilon_3 u_3) \tag{16}$$

$$\begin{aligned}\lambda_4 &= \{C_3 l_{30} - \lambda_5 u_3\sin\phi - \lambda_6 l_{30}[\tan\gamma\cos(\beta-\phi) - \sin(\beta-\phi)]\\&\quad+\lambda_7 l_3 l_{30}/(l_4 - l\cos\gamma)\}/u_3\end{aligned} \tag{17}$$

$$\lambda_5 = -\lambda_6 l_3[\tan\gamma\sin(\beta-\phi) + \cos(\beta-\phi)]/\varepsilon_3 u_3\cos\phi \tag{18}$$

$$\begin{aligned}\lambda_6 &= \big(\{(1+e\cos\theta)(\beta''-\gamma'') - 2[1+(\beta'-\gamma')]e\sin\theta\\&\quad+1.5\sin[2(\beta-\gamma)] + 2(1+e\cos\theta)(1+\beta'-\gamma')l_4'/l_4\}l_4^2\\&\quad+\lambda_7 l l_4\sin\gamma/(l_4 - l\cos\gamma)\big)\cos^2\gamma/[l + l_3\cos(\beta-\phi)]\end{aligned} \tag{19}$$

$$\begin{aligned}\lambda_7 &= (1+e\cos\theta)l_4'' - 2l_4' e\sin\theta - l_4(1+e\cos\theta)(1+\beta'-\gamma')^2\\&\quad+l_4[1 - 3\cos^2(\beta-\gamma)]\end{aligned} \tag{20}$$

The preceding expressions for λ_1, λ_2, λ_5, λ_6, and λ_7 are substituted in Eqs. (4–6). Then γ', l_4' and γ'', l_4'' are obtained in terms of α', β', l' and α'', β'', l'', α', β', l', respectively, by carrying out differentiation of Eqs. (1) and are substituted in Eqs. (4–6). The resulting equations are the equations of motion of the proposed tether system with α, β, and l as independent coordinates or degrees of freedom of the system. The equations of motion with the constraint equations for simplified cases of one-tether and two-tether systems[2,7,10–12] can be obtained from the preceding equations of motion with constrained equations by applying the conditions $\hat{a} = l_{30} = 0$, $C = C/2$, and $l_{30} = 0$, respectively.

Stability Analysis

The stability analysis is undertaken with a view to studying the nature of motion-in-the-small around the nominal system equilibrium configuration. The linearization of the governing system of equations of motion for $e = 0$ around the equilibrium position together with the consideration of finite tensions always present in the tethers in nominal equilibrium configuration and also during the motion-in-the-small, the unit functions $u_j = 1$, $j = 1, 2, 3$, and neglecting second- and higher-order terms in the perturbed variables and some suitable approximations based on an order of magnitude analysis of the coefficients lead to the following set of governing linear differential equations:

$$\begin{aligned}&(1-d_1)\alpha'' + d_1\beta'' + (-3I_r + d_2)\alpha\\&\quad-(d_2 + 3l_{30}/l_0)\beta + d_3\delta l = 0\end{aligned} \tag{21}$$

$$\begin{aligned}&\left(d_1\big/l_{40}^2\right)\alpha'' + \left(1 - d_1\big/l_{40}^2\right)\beta'' - \left(d_2\big/l_{40}^2\right)\alpha\\&\quad+\left(3 + d_2\big/l_{40}^2\right)\beta + d_4\delta l = 0\end{aligned} \tag{22}$$

$$d_5\delta l'' + d_6\delta l = 0 \tag{23}$$

where

$$d_1 = \frac{4\hat{a}^4 l_{30}^2 C^2}{9 l_{40}^2 l_0^4}, \qquad d_2 = \left(3 l_{40}\hat{b} + \frac{2\hat{a}^2 C}{l_0}\right), \qquad d_3 = \frac{4 l_{30}\hat{a}^2 C^2}{9 l_{40}^2 l_0^3}$$

$$d_4 = -\frac{4 l_{30}\hat{a} C^2}{9 l_{40}^2 l_0^2}, \qquad d_5 = \left(1 + \frac{l_{30}}{l_0}\right)\left(1 + \frac{2 l_{30}}{l_0}\right), \qquad d_6 = \frac{2C}{l_0}$$

The characteristic equation for the system now takes the form

$$\left(S^2 + \frac{d_6}{d_5}\right)\Bigg\{S^4 + \frac{1}{d_1 + d_1/l_{40}^2 - 1}\Bigg[-(-3I_r + d_2)\left(1 - \frac{d_1}{l_{40}^2}\right)$$

$$- \frac{d_1 d_2}{l_{40}^2} - \left(d_2 + \frac{3 l_{30}}{l_0}\right)\frac{d_1}{l_{40}^2} - \left(3 + \frac{d_2}{l_{40}^2}\right)(1 - d_1)\Bigg]S^2$$

$$+ \frac{1 + d_1\left[(2d_1 - 1)/l_{40}^2 - 1\right]}{\left(d_1 + d_1/l_{40}^2 - 1\right)^2}\Bigg[(-3I_r + d_2)\left(3 + \frac{d_2}{l_{40}^2}\right)$$

$$- \left(\frac{d_2}{l_{40}^2}\right)\left(d_2 + \frac{3 l_{30}}{l_0}\right)\Bigg]\Bigg\} = 0 \tag{24}$$

The conditions for system stability can be stated as

$$d_6 > 0$$

$$\left[-3 + 3I_r + 3d_1\left(1 - \frac{I_r + l_{30}/l_0}{l_{40}^2}\right) - d_2\left(1 + \frac{1}{l_{40}^2}\right)\right] > 0$$

$$-3I_r + d_2\left(1 - \frac{I_r + l_{30}/l_0}{l_{40}^2}\right) > 0 \tag{25}$$

On substituting the expressions for the various parameters in the preceding inequalities, the approximate conditions for stability are as follows:

$$l_{40}^2 > I_r + \frac{l_{30}}{l_0}$$

$$\left(3\hat{b} l_{40} + \frac{2\hat{a}^2 C}{l_0}\right)\left(1 - \frac{I_r + l_{30}/l_0}{l_{40}^2}\right) > 3I_r \tag{26}$$

In dimensional form the preceding conditions for stability can be expressed as

$$m_2 L_{40}^2 > I_y - I_z + I_x(L_{30}/L_0)$$

$$\left(3m_2 b L_{40} + \frac{2a^2 EA}{L_0 \Omega^2}\right)\left[1 - \frac{I_y - I_z + I_x(L_{30}/L_0)}{m_2 L_{40}^2}\right] > 3(I_y - I_z) \tag{27}$$

Results and Discussion

With a view to study the attitude performance of the proposed system, the detailed system attitude response is numerically simulated using Eqs. (1), (4–6), and (14–20) with the initial conditions: $\alpha_0 = \beta_0 = 0$, $l_0 = l_e$, $l_0' = 0$ and assuming $\hat{a}_1 = \hat{a}_2 = \hat{a}$, $\hat{b}_1 = \hat{b}_2 = \hat{b}$, and $l_{10} = l_{20} = l_{r0}$. If we take the same values of α_0' and β_0', that is, say 0.001, then the dependent variables have the initial values as $\gamma_0 = \phi_0 = 0$, $\gamma_0' = l_{40}' = 0$, $\phi_0' = 0.001$.

For known starting values of independent variables $\alpha_0, \beta_0, l_0, \alpha_0', \beta_0'$, and l_0' at each step of numerical integration, we first solve for dependent variables $\varepsilon_1, \varepsilon_2, c_\zeta, \varepsilon_3, \gamma, \phi$, and l_4 using the constraint relations (1). The substitution of these dependent variables as well as γ', l_4' and γ'', l_4'' in terms of α', β', l' and $\alpha'', \beta'', l'', \alpha', \beta', l'$, respectively, into Eqs. (14–20) enables the determination of λ_i, $i = 1, 2, \ldots, 7$. The values of the dependent variables $(\varepsilon_1, \varepsilon_2, c_\zeta, \varepsilon_3, \gamma, \phi, l_4), \gamma', l_4', \gamma'', l_4''$ and λ_i, $i = 1, 2, \ldots, 7$ thus explicitly available are utilized to compute the new values of the variables at the end of the step through integration of the set of the differential equations (4–6). The integration is carried out using the International Mathematical and Statistical Library routine DDASPG based on the Petzoid–Gear backward differentiation formula method. To validate the accuracy of simulation results of the proposed system, the results for simplified cases of one-tether and two-tether systems have been obtained. It was found that they fully match the results already published in the literature.[2,7,10–12]

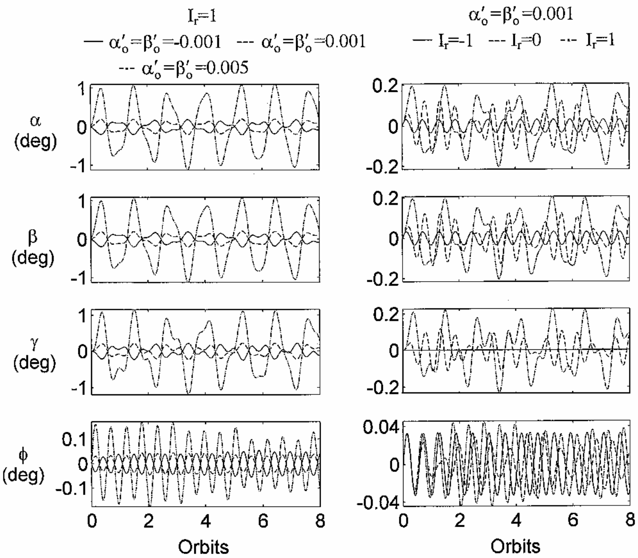

Fig. 2 Typical system librational response as affected by pitching rates and satellite mass distribution I_r: $e = 0$, $C = 7\times10^5$, $l_{r0} = 0.2$, $l_{30} = 5$, and $\hat{a} = \hat{b} = 0.04$.

Figure 2 shows the effects of initial pitching rates and satellite mass distribution parameter I_r on system librational response. Regardless of the nature of initial pitching rates, the system remains stable with low amplitude of motion in all cases considered. The stable characteristics of the system may be caused by the generation of control moments through differential changes in tether tensions as the satellite drifts. In general, the increase of initial pitching rate disturbances leads to an increase in system librational amplitudes, as expected. The order of increase in the amplitudes of all librational angles follows the same trend, except angle ϕ, which has a relatively small increase in amplitude. With regard to the effect of I_r on system librational response, it is observed that, for the case of a satellite with the most stable mass distribution in terms of gravity gradient signified by $I_r = -1$, the system librational angles α, β, and ϕ have the same stable and low amplitudes of oscillation with the maximum value of 0.033 deg. However, the librational angle γ remains almost zero. As satellite mass distribution parameter I_r changes from -1 to the most unstable satellite distribution signified by 1, the system librational angles α, β, and γ have the same stable and high amplitudes of oscillation with the maximum value of 0.24 deg. However, the librational angle ϕ has relatively low amplitudes of oscillations with the maximum value of 0.046 deg. The change of behavior of librational angles γ and ϕ can be attributed to the change of gravitational moment from that in the preceding cases.

The effects of tether lengths l_{r0} and l_{30} on system librational response are shown in Fig. 3. As tether length l_{r0} is increased from $l_{r0} = 0.2$ to 1, the maximum amplitudes of librational oscillations of α and β decrease considerably from 0.24 to 0.05 deg. The amplitudes of librational oscillations of γ get further reduced with maximum value as 0.016 deg. However, there are very small decrease in the maximum amplitudes of oscillation of angle ϕ, that is, 0.046–0.033 deg. In general, it is observed that an increase in tether length l_{r0} results in progressively reduced system librational amplitudes. This behavior of the system librational response can be caused by generation of larger restoring moments as l_{r0} increases when satellite drifts. From the stability conditions (26) the critical tether length $(l_{r0})_{cr} = 0.16$. The simulation agrees with this condition; below $(l_{r0})_{cr} = 0.16$ the attitude errors become very large and a further decrease in tether length results in instability of the system. The increase of tether length l_{30} from $l_{30} = 5$ to 30 has the similar

Table 1 Critical tether length requirements for various satellite systems ($\hat{a} = \hat{b} = 0.04$)

System data	Lightweight satellites	Medium-weight satellites	Heavyweight satellites
m_1, kg	500	1000	10^5
m_2, kg	5	25	800
I_x, kg-m^2	100	500	10^7
I_y, kg-m^2	315	1000	10^8
I_z, kg-m^2	215	500	9×10^7
L_{ref}, m	4.5	4.5	115
$(L_{r0} + L_{30})_{\text{cr}}$, m	18	18	460

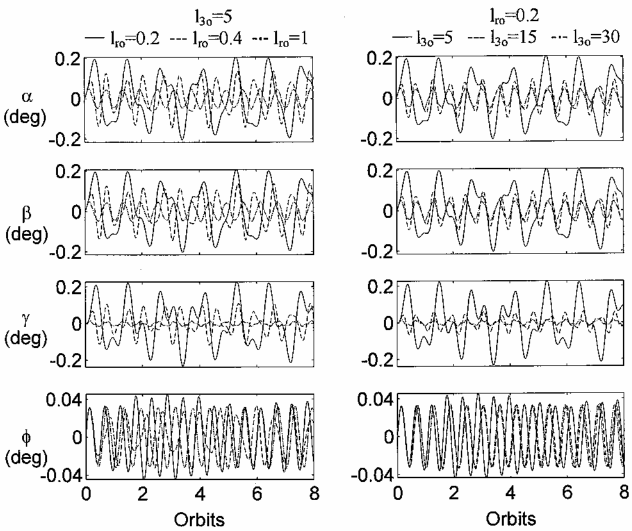

Fig. 3 Effects of varying tether lengths on system librational response: $e = 0$, $C = 7 \times 10^5$, $I_r = 1$, $\hat{a} = \hat{b} = 0.04$, and $\alpha'_0 = \beta'_0 = 0.001$.

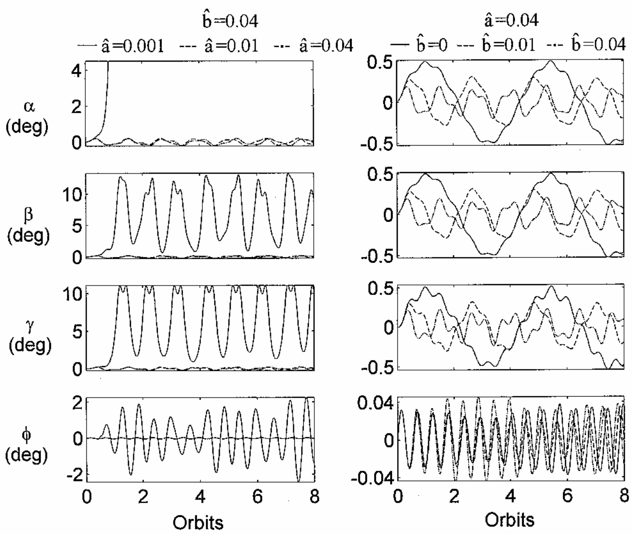

Fig. 4 Effects of varying tether offsets on system librational response: $e = 0$, $C = 7 \times 10^5$, $I_r = 1$, $l_{r0} = 0.2$, $l_{30} = 5$, and $\alpha'_0 = \beta'_0 = 0.001$.

effect on system librational response as the increase of tether length l_{r0} explained earlier. The critical tether length l_{30} obtained from the stability condition is $(l_{30})_{\text{cr}} = 4.1$, which agrees with the simulation results also. The critical or minimum tether length requirements for various combinations of satellites based on the numerical simulations are listed in Table 1.

Figure 4 shows the effects of horizontal and vertical tether offsets, that is, $\hat{a}$ and $\hat{b}$ on system librational response. When the horizontal tether offset $\hat{a}$ is a very low 0.001, the satellite librational amplitude is large and unacceptable. From the stability conditions we get $(\hat{a})_{\text{cr}} = 0.0007$. As $\hat{a}$ is increased from 0.001 to 0.01, the system

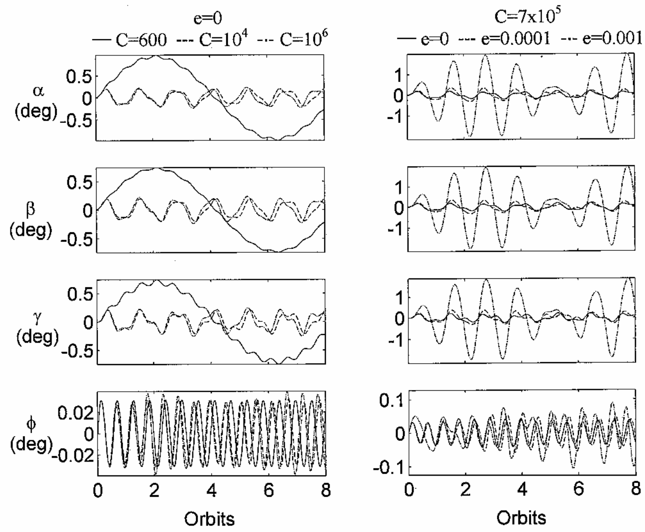

Fig. 5 Typical satellite system response as affected by parameter C and orbital eccentricity: $I_r = 1$, $l_{r0} = 0.2$, $l_{30} = 5$, $\hat{a} = \hat{b} = 0.04$, and $\alpha'_0 = \beta'_0 = 0.001$.

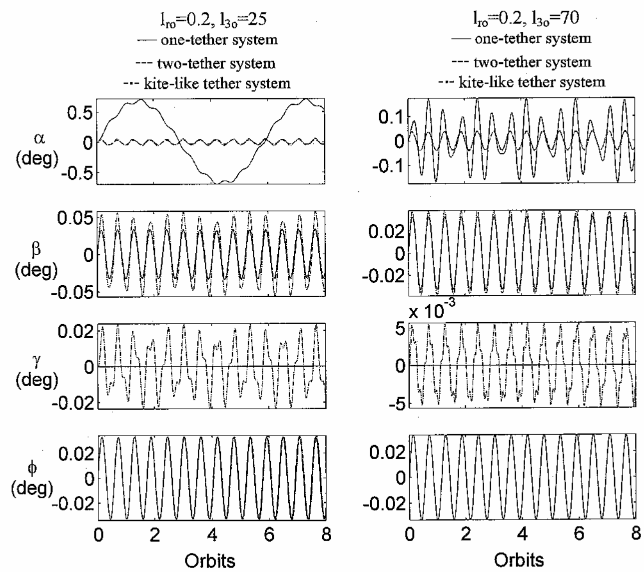

Fig. 6 Comparison of satellite attitude performance among the one-tether system, the two-tether system, and the kite-like tether system: $e = 0$, $C = 7 \times 10^5$, $I_r = 1$, $\hat{a} = \hat{b} = 0.04$, and $\alpha'_0 = \beta'_0 = 0.001$.

becomes stable with low amplitudes of librational response. Further increase in $\hat{a}$ does not result in much decrease in amplitudes of librational response. In case of vertical offset $\hat{b}$, it is observed that increasing $\hat{b}$ in general leads to improved system attitude behavior. From a practical standpoint it might be desirable to design for the largest feasible vertical offsets.

Next, we examine the effects of parameter C and orbital eccentricity e on system librational response (Fig. 5). When C has the low value of 600, the system remains stable with librational amplitudes as $|\alpha|_{\max} = 0.96$ deg, $|\beta|_{\max} = 0.75$ deg, $|\gamma|_{\max} = 0.75$ deg, and $|\phi|_{\max} = 0.032$ deg. This case is the only situation in which the amplitudes of oscillations of librational angle α differ from amplitudes of librational angle β. The change in the behavior of α and β responses as well as relatively high amplitudes of librational oscillations can be because of tether being more flexible. As C is increased to 10^6, the system librational amplitudes decrease. In case of the effect of orbital eccentricity e, even with orbital eccentricity as high as 0.001, the system remains stable with high amplitudes of librational oscillations as $|\alpha|_{\max} = |\beta|_{\max} = |\gamma|_{\max} = 2.14$ deg and $|\phi|_{\max} = 0.13$ deg.

Finally, we compare satellite attitude performance among one-tether, two-tether, and kite-tether systems (Fig. 6). To have a one-tether system, we assume $\hat{a} = l_{30} = 0$ and $C = C/2$ in the proposed system configuration. The conditions of one-tether system stability obtained from the stability conditions given by Eqs. (26) are as follows:

$$l_0^2 > I_r, \qquad l_0 > I_r/\hat{b} \tag{28}$$

In the present configuration assuming $I_r = 1$ and $\hat{b} = 0.04$, we get $(l_{r0})_{cr} = 25$. The numerical simulation of the system also satisfies this condition of stability. To have a stable system response, we consider $l_{r0} = 25.2$ for the one-tether system. The two-tether system is obtained if we consider $l_{30} = 0$ in the present configuration. The stability conditions of the two-tether system obtained from Eqs. (26) are

$$l_0^2 > I_r, \qquad \left(3\hat{b}l_0 + 2\hat{a}^2C/l_0\right)\left(1 - I_r/l_0^2\right) > 3I_r \tag{29}$$

For a stable response of the two tether-system with the case of $I_r = 1$ considered here, we should have $(l_{r0})_{cr} = 1$. In a kite-like tether system we have a choice in selecting l_{r0} and l_{30} with the restriction of these satisfying the stability conditions (26). If we take $l_{r0} = 1$, we get $(l_{30})_{cr} = 0$. This states that the kite-like tether system will be stable even with $l_{30} = 0$ for a given $l_{r0} = 1$. In other words, the kite-like tether system reduces to the two-tether system. The increase in l_{30} from its critical value of zero will add to the system stability margin and thus will improve the system attitude performance. To compare the performances of the one-tether system, the two-tether system, and the kite-like tether system, we assume $l_{r0} = 25.2$ for one-tether and two-tether systems, and $l_{r0} = 0.2$ and $l_{30} = 25$ for the kite-like tether system. The one-tether system has the highest maximum amplitudes of librational angles α and β as 0.71 and 0.034 deg, respectively, and the two-tether system has the lowest maximum amplitudes of librational angles α and β as 0.036 and 0.033 deg, respectively. The kite-like tether system's performance is close to that of a two-tether system with $|\alpha|_{max} = |\beta|_{max} = 0.058$ deg, $|\gamma|_{max} = 0.024$ deg, and $|\phi|_{max} = 0.034$ deg. When $(l_{r0})_{cr}$ is taken as 70.2 for a one-tether and a two-tether system and $(l_{r0})_{cr} = 0.2$ and $l_{30} = 70$ for a kite-like tether system, the kite-like tether system has the best performance with $|\alpha|_{max} = |\beta|_{max} = 0.039$ deg, $|\gamma|_{max} = 0.056$ deg, and $|\phi|_{max} = 0.033$ deg, followed by a two-tether system with $|\alpha|_{max} = 0.04$ deg, $|\beta|_{max} = 0.033$ deg, and one-tether system with $|\alpha|_{max} = 0.17$ deg and $|\beta|_{max} = 0.033$ deg. Thus, the attitude performance of a kite-like tether system is superior to the one-tether system and close to the two-tether system performance with longer tether lengths.

Conclusions

An analysis of attitude dynamics of a satellite using a kite-like tether configuration is presented in this paper. Linearized stability analysis undertaken here leads to several inequality constraints on system parameters. These facilitate a judicious choice of the various system parameters for the inherently stable design. Results of exact numerical integrations of the governing nonlinear equations of motion along with the stability conditions indicate that the proposed concept is feasible. The satellite attitude stabilization performance attained using the proposed system is superior to that of a one-tether system and is close to that of a two-tether system with longer tether lengths. The passive nature of the proposed system may make it particularly interesting to investigators planning future space applications.

Acknowledgment

The authors wish to acknowledge the support for this study provided by the Japan Society for the Promotion of Science.

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